Quiz A : Chapter 1

1. a
2. b
3. d
4. a
5. c
6. b
7. c
8. b

Question 9

$S(t) = 3t^4 + 7t^2$

(a) Average Velocity at interval $[1, 2]$

$S(1) = 3 + 7 = 10$

$S(2) = 3(2^4) + 7(2^2) = 76$

$$\text{Average Velocity} = \frac{76 - 10}{1} = 66 \text{ ft/s}$$

(b) Instantaneous velocity at $t = 2$

$$V = \frac{ds}{dt} = 12t^3 + 14t$$

$V(2) = 12(2^3) + 14(2)$

$= 124 \text{ ft/s}$

Question 10

$$f(x) = \frac{6}{3-x}$$

(a) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{6}{3-(x+h)} - \frac{6}{(3-x)}}{h}$$

$$\lim_{h \to 0} \frac{6\cancel{h}}{(-x+3)(3-x-h} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{6}{(3-x)(3-x-h)}$$

$$= \frac{6}{(3-x)(3-x-0)} \qquad = \frac{6}{(3-x)^2}$$

(b) at $x = 2$

$$\text{Slope} = \frac{6}{(3-2)^2} = 6$$

(c) equation of tangent.

$f(2) = \frac{6}{3-2} = 6$; Point $= (2, 6)$, slope $= 6$

$y - y_0 = m(x - x_0)$;

$y - 6 = 6(x-2)$

$$y - 6 = 6x - 12$$

equation: $y = 6x - 6$

Question 11

(a) $\lim_{x \to 0} f(x) = 4$

(b) $\lim_{x \to 1^-} f(x) = 4$

(c) $\lim_{x \to 1^+} f(x) = 2$

(d) $\lim_{x \to 1} f(x) = DNE$

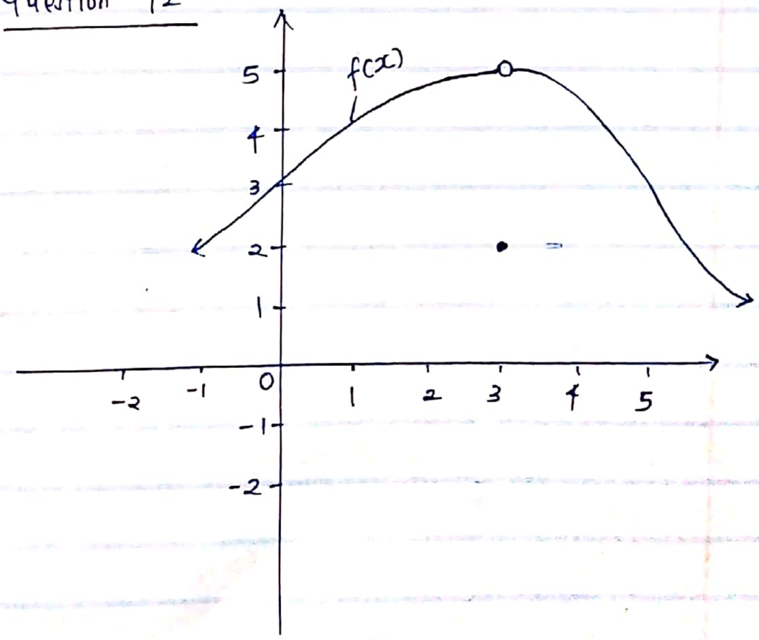
(e) $f(1) = 3$

Question 12

$f(3) = 2$

$\lim_{x \to 3^-} f(x) = 5$

$\lim_{x \to 3^+} f(x) = 5$



Question 13

(a) $\lim_{x \to 1296} \dfrac{\sqrt[4]{x} - 6}{\sqrt{x} - 36}$

Take $\dfrac{\sqrt[4]{x} - 6}{\sqrt{x} - 36} \times \dfrac{\sqrt{x} + 36}{\sqrt{x} + 36} = \dfrac{(\sqrt[4]{x} - 6)(\sqrt{x} - 36)}{x - 1296}$

$\Rightarrow$ Apply L'Hopital's rule:

$= \lim_{x \to 1296} \dfrac{\frac{d}{dx}(\sqrt[4]{x} - 6)}{\frac{d}{dx}(\sqrt{x} - 36)}$

$= \lim_{x \to 1296} \dfrac{1}{2\sqrt[4]{x}}$

$= \dfrac{1}{2\sqrt[4]{1296}}$

$= \dfrac{1}{12}$

(b) $\lim_{x \to 0} \dfrac{\sqrt{x+5} - \sqrt{5-x}}{x}$

$\dfrac{d}{dx}\left(\sqrt{x+5} - \sqrt{5-x}\right) = \dfrac{d}{dx}(x+5)^{1/2} - \dfrac{d}{dx}(5-x)^{1/2}$

$= \dfrac{1}{2}(x+5)^{-1/2} + \dfrac{1}{2}(5-x)^{1/2}$

$= \dfrac{1}{2\sqrt{x+5}} + \dfrac{1}{2\sqrt{5-x}}$

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$$\lim_{x \to 0} \frac{\sqrt{x+5} - \sqrt{5-x}}{x} = \lim_{x \to 0} \frac{1}{2\sqrt{x+5}} + \lim_{x \to 0} \frac{1}{2\sqrt{5-x}}$$

$$= \frac{1}{2\sqrt{5}} + \frac{1}{2\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}$$

(c) $$\lim_{x \to 0} \frac{4}{x}\left(\frac{2}{x+2} + \frac{2}{x-2}\right)$$

$$= \lim_{x \to 0} \frac{16}{(x+2)(x-2)} = \lim_{x \to 0} \left(\frac{16}{x+2} \cdot \frac{1}{x-2}\right)$$

$$= \frac{16}{2(-2)} = -4$$

<u>Question 14</u>

$f(x) = mx + b$

$\lim_{x \to 3} f(x) = -2$ and $\lim_{x \to 4} f(x) = 2$

$f(3) = -2$: $-2 = 3m + b$

$f(4) = 2$ $\quad 2 = 4m + b$ $-$

$-4 = -m$

$m = 4$

$-2 = 12 + b$; $b = -2 - 12 = -14$ $\therefore$ $m = 4$

$b = -14$

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Question 15

$$\lim_{x \to -10} \frac{2f(x) - x^2}{\sqrt{f(x)}} \quad \text{given} \quad \lim_{x \to -10} f(x) = 16$$

$$= \lim_{x \to -10} 2f(x) - x^2 \times \lim_{x \to -10} \frac{1}{\sqrt{f(x)}}$$

$$= \left[2\lim_{x \to -10} f(x) - \lim_{x \to -10} (x^2) \right] \times \lim_{x \to -10} \frac{1}{\sqrt{f(x)}}$$

$$= (2 \times 16) - 100 \times \frac{1}{\sqrt{16}}$$

$$= \frac{-68}{\sqrt{16}} = \frac{-17}{\cancel{4}}$$

Ans = -17

Question 16

No, it does not. The limit of a function at a particular point does not exist if one-sided limits are not equal. However, the value of the function at that point can exis and is defined

Question 17

$$g(x) = \begin{cases} x^2+2, & x<1 \\ x+2, & x \geq 1 \end{cases}$$

at $x = 1$

$x^2+2, \; x=1 \quad = \quad 1+2 = 3$

$x+2, \; x=1 \quad = \quad 1+2 = 3$

The function is continuous. The left limit and right limit at $x=1$ agree.

Question 18

$f(5) = 3$

$\lim_{x \to 5^-} f(x) = -5$

$\lim_{x \to 5^+} f(x) = -2$